

Decay constants of the charmed vector mesons D^* and D_s^* from QCD sum rules

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We present a sum-rule calculation of the decay constants of the charmed vector mesons D^* and D_s^* from the two-point correlator of vector currents. First, we show that the perturbative expansion in terms of the pole mass exhibits no sign of convergence whereas the reorganization of this expansion in terms of the $\overline{\text{MS}}$ mass leads to a distinct hierarchy. Second, making use of the operator product expansion in terms of the $\overline{\text{MS}}$ mass, we determine the decay constants of the D^* and D_s^* mesons with an emphasis on the uncertainties in these theoretically predicted quantities related both to the input QCD parameters and to the limited accuracy of the method of sum rules. Our results are $f_{D^*} = (252.2 \pm 22.3_{\text{OPE}} \pm 4_{\text{syst}})$ MeV and $f_{D_s^*} = (305.5 \pm 26.8_{\text{OPE}} \pm 5_{\text{syst}})$ MeV. For the ratios of the vector-to-pseudoscalar decay constants we report $f_{D^*}/f_D = 1.221 \pm 0.080_{\text{OPE}} \pm 0.008_{\text{syst}}$ and $f_{D_s^*}/f_{D_s} = 1.241 \pm 0.057_{\text{OPE}} \pm 0.007_{\text{syst}}$.

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1. INTRODUCTION

The extraction of the decay constants of ground-state vector mesons within the method of QCD sum rules [1, 2] is based on the analysis of the two-point correlation function

$$i \int d^4x e^{ipx} \langle 0 | T(j_\mu(x) j_\nu^\dagger(0)) | 0 \rangle = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) \Pi(p^2) + \frac{p_\mu p_\nu}{p^2} \Pi_L(p^2) \quad (1.1)$$

of the vector heavy-light currents for a heavy quark Q of mass m_Q and a light quark q of mass m

$$j_\mu(x) = \bar{q}(x) \gamma_\mu Q(x), \quad (1.2)$$

or, more precisely, on the Borel transform $\Pi(p^2) \rightarrow \Pi(\tau)$ of its transverse structure to Borel variable τ . Equating $\Pi(\tau)$ as calculated within QCD and the expression obtained by inserting a complete set of hadron states yields the sum rule

$$\Pi(\tau) = f_V^2 M_V^2 e^{-M_V^2 \tau} + \int_{s_{\text{phys}}}^{\infty} ds e^{-s\tau} \rho_{\text{hadr}}(s) = \int_{(m_Q+m)^2}^{\infty} ds e^{-s\tau} \rho_{\text{pert}}(s, \mu) + \Pi_{\text{power}}(\tau, \mu). \quad (1.3)$$

Here, M_V is the mass, f_V the decay constant, and $\varepsilon_\mu(p)$ the polarization vector of the vector meson V under study:

$$\langle 0 | \bar{q} \gamma_\mu Q | V(p) \rangle = f_V M_V \varepsilon_\mu(p). \quad (1.4)$$

For the correlator (1.1), $s_{\text{phys}} = (M_P + M_\pi)^2$ is the physical continuum threshold, wherein M_P denotes the mass of the pseudoscalar meson containing Q . For large values of τ , the ground state dominates the correlator and thus its properties may be calculated from the correlation function (1.1).

In perturbation theory, the correlation function is obtained as expansion in powers of the strong coupling constant $\alpha_s(\mu)$. The best known three-loop perturbative spectral density has been calculated in [3] in terms of the *pole* mass of the heavy quark Q (called M here) and for a massless second quark [$\alpha_s(\mu)$ is the running coupling constant in the $\overline{\text{MS}}$ scheme]:

$$\rho_{\text{pert}}(s) = \rho^{(0)}(s, M) + \frac{\alpha_s(\mu)}{\pi} \rho^{(1)}(s, M) + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \rho^{(2)}(s, M, \mu) + \dots \quad (1.5)$$

For two massive quarks, the two-loop spectral density in terms of their pole masses was obtained in [4].

However, already for the case of the pseudoscalar correlator it was found that the perturbative expansion in terms of the heavy-quark pole mass does not exhibit any sign of convergence; this problem was cured by rearranging the perturbative expansion in terms of the corresponding running $\overline{\text{MS}}$ mass [5]. We show that precisely the same happens in the case of the vector correlator (1.1).

Another subtlety — related to the truncation of the perturbative expansion — is the unphysical dependence of the obtained ground-state parameters on the renormalization scale μ : of course, the full correlator (1.1) does not depend on μ ; however, both the perturbative expansion truncated at fixed order in α_s and the truncated power corrections $\Pi_{\text{power}}(\tau, \mu)$ depend on μ . For the pseudoscalar-meson decay constants, this dependence was found to be rather mild [6]. Unfortunately, as we shall demonstrate in this analysis, for the vector-meson decay constants the μ dependence is rather pronounced; this leads to a larger corresponding error in the decay constants of vector mesons obtained from QCD sum rules.

Furthermore, the truncated operator product expansion (OPE) does not allow one to calculate the correlator for sufficiently large τ , such that the continuum states give a sizable contribution to $\Pi(\tau)$ in the corresponding τ -range. In order to get rid of the continuum contribution, the concept of duality is invoked: Perturbative-QCD spectral density $\rho_{\text{pert}}(s)$ and hadron spectral density $\rho_{\text{hadr}}(s)$ resemble each other at large values of s ; thus, for sufficiently large values of the parameter \bar{s} , (far) above the resonance region, one arrives at the duality relation

$$\int_{\bar{s}}^{\infty} ds e^{-s\tau} \rho_{\text{hadr}}(s) = \int_{\bar{s}}^{\infty} ds e^{-s\tau} \rho_{\text{pert}}(s). \quad (1.6)$$

Now, in order to express the continuum contribution in terms of the perturbative contribution, this relationship should be extended down to the hadronic threshold s_{phys} . However, the spectral densities $\rho_{\text{pert}}(s)$ and $\rho_{\text{hadr}}(s)$ are obviously different in the region near s_{phys} . Therefore, one can only expect to obtain a relation of the form

$$\int_{s_{\text{phys}}}^{\infty} ds e^{-s\tau} \rho_{\text{hadr}}(s) = \int_{s_{\text{eff}}(\tau)}^{\infty} ds e^{-s\tau} \rho_{\text{pert}}(s), \quad (1.7)$$

where $s_{\text{eff}}(\tau)$ is different from the physical threshold s_{phys} . Obviously, for the same reason which causes $s_{\text{eff}}(\tau) \neq s_{\text{phys}}$, $s_{\text{eff}}(\tau)$ must be a function of the parameter τ [7, 8]. By virtue of (1.7), we may rewrite the sum rule (1.3) as

$$f_V^2 M_V^2 e^{-M_V^2 \tau} = \int_{(m_Q+m)^2}^{s_{\text{eff}}(\tau)} ds e^{-s\tau} \rho_{\text{pert}}(s, \mu) + \Pi_{\text{power}}(\tau, \mu) \equiv \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)). \quad (1.8)$$

We refer to the right-hand side of this equation as the *dual correlator* and to the τ -dependent effective threshold that corresponds to the true values of the ground-state parameters in the left-hand side of (1.8) as the *exact effective threshold*; by definition, the exact effective threshold makes Eq. (1.8) an identity. One essential property of the exact effective threshold should be mentioned: Whereas the exact correlation function and its truncated OPE have very different energy dependences in the Minkowski space, after performing the Borel transform $\Pi(p^2) \rightarrow \Pi(\tau)$, the complicated energy dependence of the exact correlation function leads to only a weak τ -dependence of the exact effective threshold. This feature opens the possibility to find realistic approximations to this exact τ -dependent threshold and to obtain in this way reliable estimates for the bound-state parameters.

Obviously, the exact effective threshold is unknown. Thus, the extraction of the decay constant requires, in addition to $\rho_{\text{pert}}(s, \mu)$ and $\Pi_{\text{power}}(\tau, \mu)$, as further input, a criterion for obtaining an approximation to the exact effective threshold. In [8] we developed the algorithm for fixing $s_{\text{eff}}(\tau)$ which allows one to reliably extract the ground-state parameters on the basis of (i) an accurate OPE for the Green functions and (ii) the known value of the ground-state mass.

We shall demonstrate that QCD sum rules armed with this algorithm allow a very satisfactory extraction of the vector-meson decay constants, with an accuracy that is certainly competitive to that found using lattice QCD.

2. OPERATOR PRODUCT EXPANSION AND CHOICE OF SCHEME FOR HEAVY-QUARK MASSES

We start with the OPE for the correlation function (1.1). We may use the perturbative spectral density $\rho_{\text{pert}}(s, M)$ of [3] in terms of the pole mass of the heavy quark. An alternative option is to reorganize the perturbative expansion in terms of the running $\overline{\text{MS}}$ mass; the relevant analytic expressions are given in [9], see also the discussion in the Appendix.

Figure 1 illustrates the sum-rule estimates for f_{D^*} arising from (1.8) for these two choices of the c -quark mass: the pole mass M_c and the running $\overline{\text{MS}}$ mass $\overline{m}_c(\mu)$. The numerical OPE-parameter values entering this game

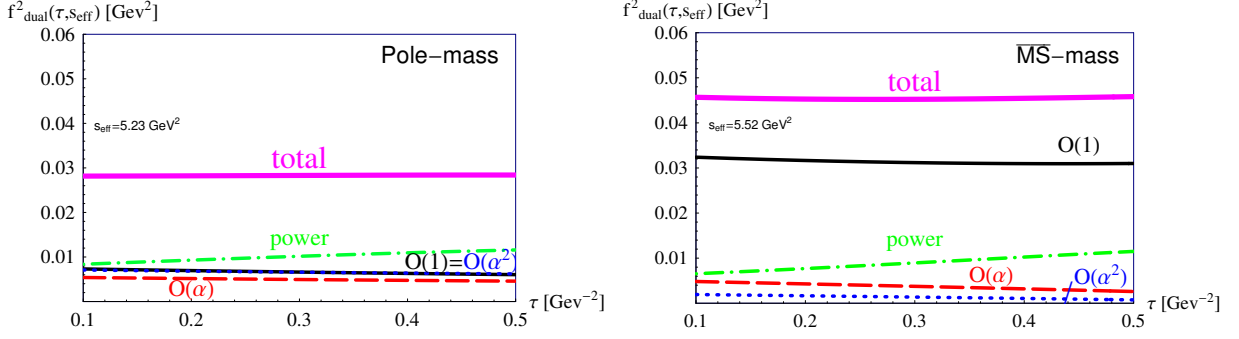


Fig. 1: QCD sum-rule estimates for f_{D^*} extracted by expressing the OPE in terms of the c -quark pole mass (left) or $\overline{\text{MS}}$ mass (right). The pole mass $M_c = 1.699$ GeV, used in the left plot, has been recalculated by the $O(\alpha_s^2)$ relation (A.2) from the running $\overline{\text{MS}}$ mass $m_c(m_c) = 1.279$ GeV, used in the right plot. For each case separately, a constant effective continuum threshold s_{eff} is determined by requiring “maximal stability” of the obtained decay constant in the Borel window $0.1 \leq \tau (\text{GeV}^{-2}) \leq 0.5$. As a result, s_{eff} turns out to be different for the two schemes: $s_{\text{eff}} = 5.23$ GeV² for the pole-mass scheme (left), $s_{\text{eff}} = 5.52$ GeV² for the $\overline{\text{MS}}$ scheme (right). Bold lines — total results, solid lines (black) — $O(1)$ contributions; dashed lines (red) — $O(\alpha_s)$ contributions; dotted lines (blue) — $O(\alpha_s^2)$ contributions; dot-dashed lines (green) — power contributions.

read [5, 6, 10, 11]

$$\begin{aligned} \overline{m}_c(\overline{m}_c) &= (1.275 \pm 0.025) \text{ GeV}, \quad \overline{m}(2 \text{ GeV}) = (3.42 \pm 0.09) \text{ MeV}, \quad \overline{m}_s(2 \text{ GeV}) = (93.8 \pm 2.4) \text{ MeV}, \\ \alpha_s(M_Z) &= 0.1184 \pm 0.0020, \\ \langle \bar{q}q \rangle(2 \text{ GeV}) &= -((267 \pm 17) \text{ MeV})^3, \quad \langle \bar{s}s \rangle(2 \text{ GeV}) / \langle \bar{q}q \rangle(2 \text{ GeV}) = 0.8 \pm 0.3, \quad \left\langle \frac{\alpha_s}{\pi} GG \right\rangle = (0.024 \pm 0.012) \text{ GeV}^4. \end{aligned} \quad (2.1)$$

The pole mass, recomputed from the $O(\alpha_s^2)$ relation between \overline{m}_c and M_c [12], reads $M_c = 1.699$ GeV. The sum-rule estimates shown in Fig. 1 are obtained for a τ -independent effective threshold s_{eff} . Its values, different for pole-mass OPE and $\overline{\text{MS}}$ -mass OPE, are found by requiring maximal stability of the extracted decay constant in the chosen Borel window (as detailed in Sect. 3). Let us emphasize that, for the moment, a *constant* effective threshold and the stability criterion for determining its numerical value are adopted only for illustration: As we have demonstrated in many examples [7], using a constant effective threshold provides rather inaccurate estimates for the decay constant and does not allow one to probe the systematic error of this extraction.

Nevertheless, the results of Fig. 1 illustrate some of the essential features of the extraction procedures. First, using the pole-mass OPE, one observes no hierarchy of the perturbative contributions to the dual correlator – the $O(1)$, $O(\alpha_s)$, and $O(\alpha_s^2)$ contributions have the same size. Obviously, there is no reason to expect the unknown higher-order perturbative corrections to be small; the pole-mass OPE truncated at order $O(\alpha_s^2)$ and the corresponding ground-state parameters suffer from large uncertainties. On the other hand, reorganizing the perturbative expansion in terms of the $\overline{\text{MS}}$ mass of the heavy quark leads to a clear hierarchy and allows a reliable extraction of the ground-state parameters. This is precisely the same feature that has been observed for the pseudoscalar correlator.

Second, there is a huge numerical difference between the decay constants obtained using the pole-mass OPE and the running-mass OPE if one compares calculations obtained for the values of $\overline{m}_c(\overline{m}_c)$ and its pole-mass $O(\alpha_s^2)$ counterpart given above. However, comparing the results of the truncated pole-mass and running-mass OPE requires some caution, as the perturbative expansion of the pole mass in terms of the running mass displays its asymptotic nature already at lowest orders [12]: $M_c = \overline{m}_c(\overline{m}_c)(1 + 1.33a + 10.32a^2 + 116.50a^3)$, with $a = \alpha_s(\overline{m}_c)/\pi = 0.126 \pm 0.002$. Assigning the uncertainty of the pole-mass value that corresponds to a specific running-mass value as, e.g., the size of the last included term in the perturbative relation, in our case of the $O(\alpha_s^2)$ term, amounts to a 15% uncertainty in M_c . Due to a large sensitivity of the extracted decay constant to the precise value of the charm-quark mass, the uncertainty of 15% in M_c leads to a 100% uncertainty in the dual pole-mass correlator. With such an uncertainty, the results obtained from the pole-mass and the running-mass OPE in Fig. 1 are compatible with each other, but suggest that the accuracy of the $O(\alpha_s^2)$ -truncated pole-mass OPE is rather bad.

We therefore make use of the OPE in terms of the running $\overline{\text{MS}}$ mass for the analysis of f_V . Accordingly, henceforth the quark masses m_Q and m , and the strong coupling α_s denote the $\overline{\text{MS}}$ running quantities.

3. EXTRACTION OF THE DECAY CONSTANTS

In order to extract the decay constants from our QCD sum rule, we first have to fix the working τ -window where the OPE provides a sufficiently accurate description of the exact correlator (i.e., all higher-order radiative and power corrections are under control) and the ground state gives a “sizable” contribution to the correlator. We shall adopt the window fixed in our previous analysis of the decay constants of the D and D_s mesons [6].

Next, we must fix the effective continuum threshold $s_{\text{eff}}(\tau)$. The corresponding algorithm was developed and verified in quantum-mechanical potential models [8, 13] and proven to work successfully for the decay constants of the heavy pseudoscalar mesons [14].

We define the *dual invariant mass* M_{dual} and the *dual decay constant* f_{dual} by

$$M_{\text{dual}}^2(\tau) \equiv -\frac{d}{d\tau} \log \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)), \quad f_{\text{dual}}^2(\tau) \equiv M_V^{-2} e^{M_V^2 \tau} \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)). \quad (3.1)$$

For a properly constructed $\Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau))$, the dual mass coincides with the actual ground-state mass M_V . Therefore, any deviation of the dual mass from M_V is an indication of the contamination of the dual correlator by excited states.

For any trial functional form of the effective threshold, one obtains a variational solution by minimizing the difference between the dual mass (3.1) and the actual (experimental) mass in the Borel window. This variational solution provides the decay constant then via (3.1). We consider a set of τ -dependent Ansätze for the effective continuum threshold, viz.,

$$s_{\text{eff}}^{(n)}(\tau) = \sum_{j=0}^n s_j^{(n)} \tau^j, \quad (3.2)$$

and fix the parameters on the right-hand side of (3.2) by minimizing

$$\chi^2 \equiv \frac{1}{N} \sum_{i=1}^N [M_{\text{dual}}^2(\tau_i) - M_V^2]^2 \quad (3.3)$$

over the window. This gives us the coefficients $s_j^{(n)}$ of the effective continuum threshold and thus eventually the decay constant f_V . Still, different Ansätze for $s_{\text{eff}}(\tau)$ yield different predictions for the decay constant.

A detailed analysis of quantum-mechanical models for different potentials indicated that it is sufficient to consider polynomials up to third order: In this case, the band delimited by the results obtained for linear, quadratic, and cubic Ansätze for $s_{\text{eff}}(\tau)$ contains the true value of the decay constant. Even the good knowledge of the truncated OPE does not allow to determine the decay constant precisely, but it allows us to provide the range of values containing the true value of this decay constant. The width of this range may be then treated as a *systematic error* related to a principally limited accuracy of the method. Presently, we do not see other possibilities to obtain a more reliable estimate for the systematic error. Noteworthy, considering a merely τ -independent threshold does not allow one to probe the accuracy of the obtained estimate for f_V .

On top of the systematic error comes the *OPE-related error* of the decay constant: the OPE parameters are known with some errors, inducing a corresponding error of f_V . This OPE-related (or statistical) error is determined by averaging the results for the decay constant assuming for the OPE parameters Gaussian distributions with the central values and standard deviations quoted in (2.1) and a flat distribution over the scale μ in the range $1 < \mu$ (GeV) < 3 .

A. Decay constant of the D^* meson

Following [6], we choose for the τ -window for the charmed mesons the interval $\tau = (0.1-0.5) \text{ GeV}^{-2}$. Figure 2 shows the application of our procedure for fixing the effective continuum threshold and extracting the resulting f_{D^*} . As must be obvious from Fig. 2a, using a constant threshold leads to a contamination of the dual correlator by excited states (at a percent level in the dual mass) while this contamination is strongly reduced for $n > 0$. The results for the decay constant in Fig. 2b corresponding to $n > 0$ are nicely grouped together, whereas the $n = 0$ prediction lies $\approx 30 \text{ MeV}$ below. Interestingly, the effect visible at only a 1–2% level in the dual mass in Fig. 2a manifests itself at a 10% level in the decay constant in Fig. 2b. Consequently, the results obtained for $n > 0$, less contaminated by excited states, constitute a significant improvement with respect to the results obtained for a constant threshold, i.e., $n = 0$. Allowing the effective threshold to depend on τ brings the QCD sum-rule results into agreement with the recent lattice finding $f_{D^*} = (278 \pm 13 \pm 10) \text{ MeV}$ [15].

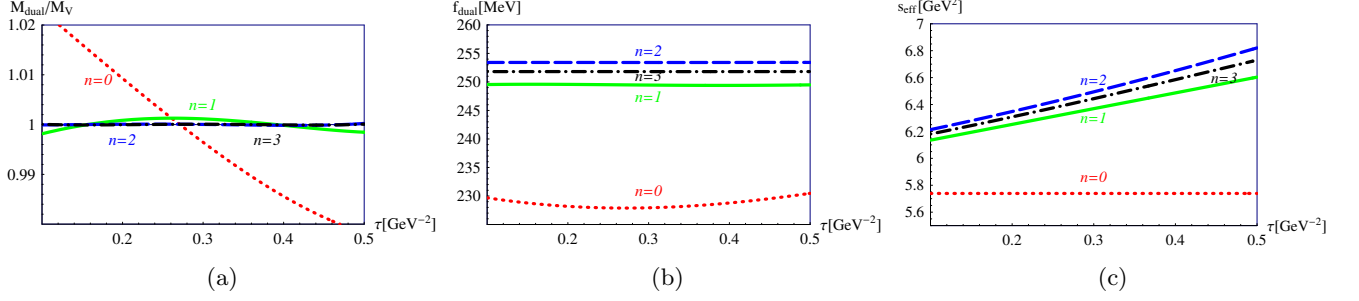


Fig. 2: Dependence on the Borel parameter τ of the dual mass (a) and the dual decay constant (b) of the D^* meson, obtained by employing different Ansätze (3.2) for the effective continuum threshold $s_{\text{eff}}(\tau)$ and fixing all thresholds according to (3.3); the results are presented for central values of all OPE parameters and for an average scale $\mu = \mu^* = 1.84$ GeV, where the average scale μ^* is defined by (3.6). (c) Our τ -dependent effective thresholds obtained by the fitting procedure as explained in the text. The integer $n = 0, 1, 2, 3$ is the degree of the polynomial in our Ansatz (3.2) for $s_{\text{eff}}(\tau)$: dotted lines (red) — $n = 0$; solid lines (green) — $n = 1$; dashed lines (blue) — $n = 2$; dot-dashed lines (black) — $n = 3$.

The dependence of the extracted f_{D^*} on both c -quark mass $m_c \equiv \overline{m}_c(\overline{m}_c)$ and quark condensate $\langle \bar{q}q \rangle \equiv \langle \bar{q}q(2 \text{ GeV}) \rangle$ at the average scale $\mu^* = 1.84$ GeV (see (3.7) below) may be parameterized as

$$f_{D^*}^{\text{dual}}(\mu = \mu^*, m_c, \langle \bar{q}q \rangle) = \left[252.2 - 10 \left(\frac{m_c - 1.275 \text{ GeV}}{0.025 \text{ GeV}} \right) + 6 \left(\frac{|\langle \bar{q}q \rangle|^{1/3} - 0.267 \text{ GeV}}{0.01 \text{ GeV}} \right) \pm 4_{(\text{syst})} \right] \text{ MeV}. \quad (3.4)$$

The extracted value of f_{D^*} turns out to be very sensitive to the choice of the renormalization scale μ . Recall once more that this dependence is unphysical and induced by the truncation of the perturbation series. The μ dependence of f_{D^*} for the central values of the other OPE parameters is depicted in Fig. 3a. For each μ , the value of f_{D^*} (and f_D) corresponds to the average of the interval formed by the results obtained from the linear, quadratic, and cubic Ansätze for the effective continuum threshold. It should be noted that the dependence of f_{D^*} on μ is clearly nonlinear. The

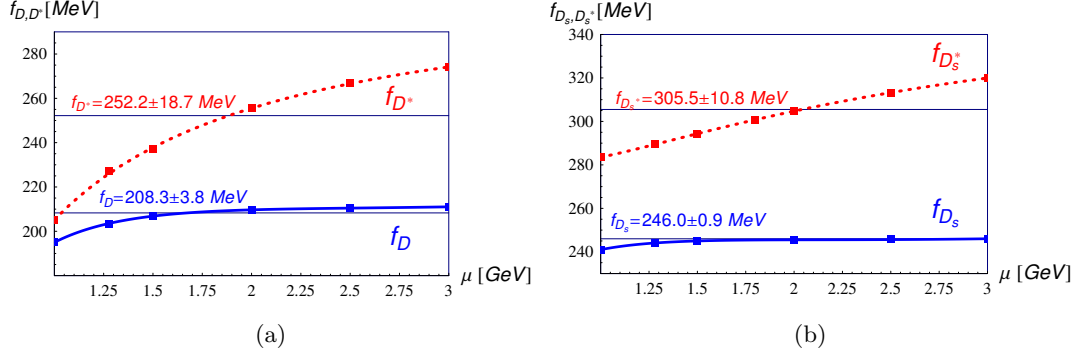


Fig. 3: Dependence on μ of the dual decay constants: (a) $f_{D^*}^{\text{dual}}(\mu)$ and $f_D^{\text{dual}}(\mu)$, (b) $f_{D_s^*}^{\text{dual}}(\mu)$ and $f_{D_s}^{\text{dual}}(\mu)$. The depicted results are obtained as follows: for a fixed value of μ , central values of the OPE parameters in (2.1) and a Borel parameter τ within the window $0.1 < \tau \text{ (GeV}^{-2}\text{)} < 0.5$, we determine the effective thresholds by our procedure; the presented dual decay constant then is the average of the band formed by the linear, quadratic, and cubic Ansätze for the effective threshold. Clearly, the effective thresholds turn out to depend on the scale μ . Dotted lines (red) — vector mesons; solid lines (blue) — pseudoscalar mesons.

obtained results may be well interpolated by the following simple formula:

$$f_{D^*}^{\text{dual}}(\mu) = 252.2 \text{ MeV} \left[1 + 0.233 \log(\mu/\mu^*) - 0.096 \log^2(\mu/\mu^*) + 0.17 \log^3(\mu/\mu^*) \right], \quad \mu^* = 1.84 \text{ GeV}. \quad (3.5)$$

Here, μ^* is the average scale defined in the standard way:

$$\langle f_V^{\text{dual}}(\mu) \rangle = f_V^{\text{dual}}(\mu^*), \quad (3.6)$$

assuming a flat probability distribution for μ in the range $1 < \mu \text{ (GeV)} < 3$. The corresponding standard deviation of f_{D^*} is 18.7 MeV. For comparison, we also provide the μ dependence and the average scale μ^* for f_D from [6]:

$$f_D^{\text{dual}}(\mu) = 208.3 \text{ MeV} \left[1 + 0.06 \log(\mu/\mu^*) - 0.11 \log^2(\mu/\mu^*) + 0.08 \log^3(\mu/\mu^*) \right], \quad \mu^* = 1.62 \text{ GeV}. \quad (3.7)$$

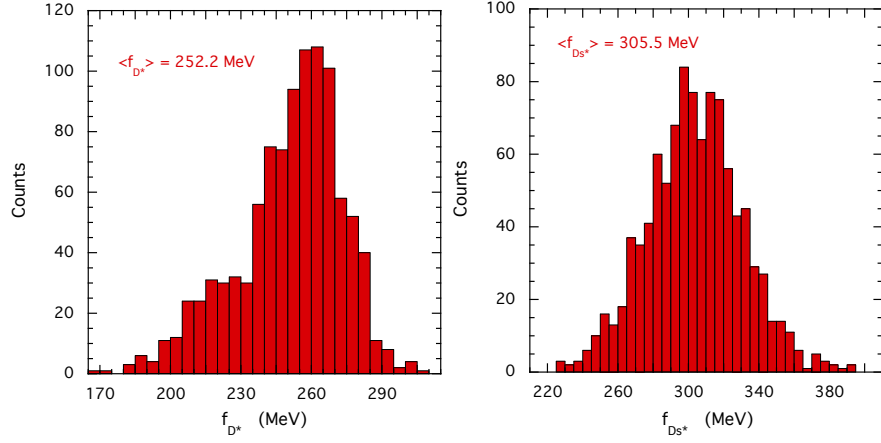


Fig. 4: Distribution of the decay constants f_{D^*} (a) and $f_{D_s^*}$ (b) obtained by 1000 bootstrap events.

Obviously, the μ dependence of the pseudoscalar correlator is much weaker. This effect has the following origin: both the truncated perturbative dual correlator $\Pi_{\text{pert}}^{\text{dual}}(s_{\text{eff}}, \tau, \mu)$ and the truncated $\Pi_{\text{power}}(\tau, \mu)$ exhibit a rather pronounced μ dependence. For the pseudoscalar correlator, these μ dependencies to a large extent cancel each other, whereas for the vector correlator the cancellation does not occur.

Assuming Gaussian distributions for all the OPE parameters collected in (2.1) and a flat μ distribution in the range $1 < \mu \text{ (GeV)} < 3$, we obtain the distribution of f_{D^*} depicted in Fig. 4. The f_{D^*} distribution is clearly not Gaussian, which is due to the nonlinear μ dependence of f_{D^*} shown in Fig. 3. For the average and the standard deviation of the D^* -meson decay constant we obtain

$$f_{D^*} = (252.2 \pm 22.3_{\text{(OPE)}} \pm 4_{\text{(syst)}}) \text{ MeV}. \quad (3.8)$$

The OPE uncertainty is composed as follows: 18.7 MeV are due to the variation of the scale μ , 10 MeV arise from the error in $m_c \equiv \overline{m}_c(\overline{m}_c)$, 2 MeV from $\alpha_s(M_Z)$, 6 MeV from the quark condensate, and 3 MeV from the gluon condensate. Higher condensates contribute less than 1 MeV to this error.

Combining our above results with those for f_D from our earlier analysis [6], we obtain

$$f_{D^*}/f_D = 1.221 \pm 0.080_{\text{(OPE)}} \pm 0.008_{\text{(syst)}}. \quad (3.9)$$

The OPE uncertainty of this ratio is fully dominated by the impact of the μ dependence.

B. Decay constant of D_s^* meson

For the D_s^* , we take the same Borel-parameter window as for D^* : $\tau = (0.1-0.5) \text{ GeV}^{-2}$. Figure 5 provides the details of our extraction procedure. Our results for the D_s^* -meson decay constant may be summarized as $[m_s \equiv \overline{m}_s(2 \text{ MeV})]$

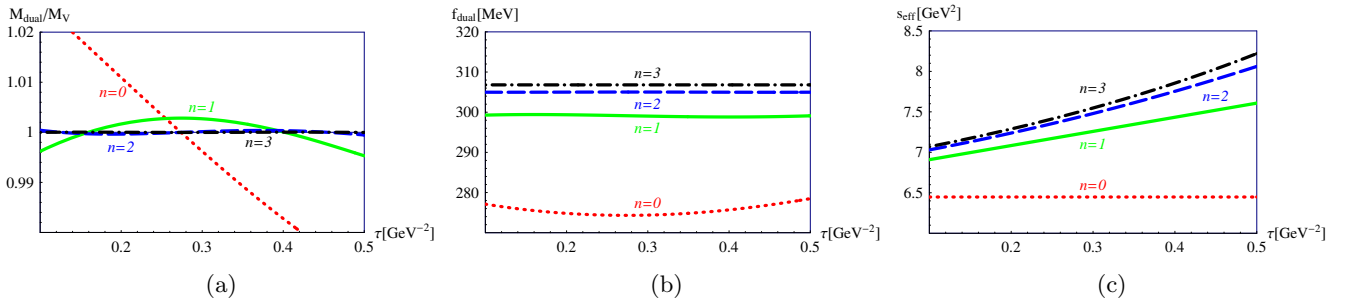


Fig. 5: Same as Fig. 2 but for the D_s^* meson, at the average renormalization scale appropriate for the D_s^* meson: $\mu^* = 1.94 \text{ MeV}$.

$$f_{D_s^*}^{\text{dual}}(\mu = \mu^*, m_c, m_s, \langle \bar{s}s \rangle) = \left[305.5 - 12.4 \left(\frac{m_c - 1.275 \text{ GeV}}{0.025 \text{ GeV}} \right) + 1.7 \left(\frac{m_s - 0.1 \text{ GeV}}{0.004 \text{ GeV}} \right) + 3.9 \left(\frac{|\langle \bar{s}s \rangle|^{1/3} - 0.248 \text{ GeV}}{0.01 \text{ GeV}} \right) \pm 5_{(\text{syst})} \right] \text{ MeV}. \quad (3.10)$$

Similarly to f_{D^*} , also the extracted decay constant of D_s^* exhibits a rather strong and almost linear μ dependence (see Fig. 3b) which, for average values of the other OPE parameters, may be parameterized as

$$f_{D_s^*}^{\text{dual}}(\mu) = 305.5 \text{ MeV} \left[1 + 0.124 \log(\mu/\mu^*) + 0.014 \log^2(\mu/\mu^*) - 0.034 \log^3(\mu/\mu^*) \right], \quad \mu^* = 1.94 \text{ GeV}. \quad (3.11)$$

For comparison, the μ dependence and the average scale μ^* for f_{D_s} from [6] is also given:

$$f_{D_s}^{\text{dual}}(\mu) = 246.0 \text{ MeV} \left[1 + 0.01 \log(\mu/\mu^*) - 0.03 \log^2(\mu/\mu^*) + 0.04 \log^3(\mu/\mu^*) \right], \quad \mu^* = 1.52 \text{ GeV}. \quad (3.12)$$

Notice that f_{D_s} is extremely stable with respect to μ . This is an effect of an almost precise cancellation between the μ dependencies of the dual perturbative and the condensate contributions.

Again, for Gaussian distributions of all OPE parameters and a flat distribution in μ in the range $1 < \mu \text{ (GeV)} < 3$, we find a nearly Gaussian distribution of $f_{D_s^*}$ in Fig. 4 which yields

$$f_{D_s^*} = (305.5 \pm 26.8_{(\text{OPE})} \pm 5_{(\text{syst})}) \text{ MeV}. \quad (3.13)$$

The composition of the OPE error reads: 10.8 MeV are due to the variation of the scale μ , 19.5 MeV are caused by the error of strange-quark condensate, 12.5 MeV by the error of $\overline{m}_c(\overline{m}_c)$, 6.4 MeV by the gluon condensate, 1.7 MeV by the strange-quark mass, and 1.4 MeV by $\alpha_s(M_Z)$. Higher condensates contribute 2 MeV to this uncertainty. Our result (3.8) is in good agreement with $f_{D_s^*} = (311 \pm 9) \text{ MeV}$ from lattice QCD [15].

Making use of our result for f_{D_s} from [6], we obtain, for the ratio of the vector and the pseudoscalar decay constants,

$$f_{D_s^*}/f_{D_s} = 1.241 \pm 0.057_{(\text{OPE})} \pm 0.007_{(\text{syst})}. \quad (3.14)$$

The OPE uncertainty in this ratio is dominated by the errors arising from the μ dependence (0.043) and the gluon condensate (0.026).

Finally, for the ratio of the D_s^* and D^* decay constants, we get

$$f_{D_s^*}/f_{D^*} = 1.211 \pm 0.061_{(\text{OPE})} \pm 0.007_{(\text{syst})}. \quad (3.15)$$

The error here arises mainly from the errors in the strange-quark mass and the condensates ratio $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.3$. The value (3.15) is slightly larger than but not in disagreement with the lattice result $f_{D_s^*}/f_{D^*} = 1.16 \pm 0.02 \pm 0.06$ [15].¹

4. SUMMARY AND CONCLUSIONS

Exploiting the tools offered by QCD sum rules, we analyzed in great detail the decay constants of charmed vector mesons, paying special attention to the involved uncertainties of the predicted decay-constant values: the OPE error (related to the precision with which the QCD parameters are known) and the systematic error intrinsic to the sum-rule approach as a whole (reflecting the limited accuracy of the extraction procedure). We thus gained important insights:

- (i) As was already noted in the case of heavy pseudoscalar mesons [6], also for the vector correlator the perturbative expansion in terms of the heavy-quark pole-mass does not seem to converge whereas reorganizing it in terms of the corresponding running mass leads to a clear hierarchy of the perturbative contributions.
- (ii) The dependence of the vector correlator, known at three-loop accuracy, on the renormalization scale μ turns out to be sizeably stronger compared to the pseudoscalar correlator. Respectively, the error related to the remaining scale dependence of the vector-meson decay constant proves to be twice as large as that for pseudoscalar-meson decay constant.

¹ For an analysis of the vector-meson decay constants within the framework of quark models, we refer to [16].

- (iii) We allowed for a Borel-parameter-dependent effective threshold for the decay-constant extractions. Obviously, such a τ -dependent effective threshold visibly improves the stability of the dual mass in the Borel window. This means that the dual correlator is much less contaminated by excited states than the one inferred upon confining oneself to τ -independent effective thresholds. We thus get, as our estimates for the vector-meson decay constants,

$$f_{D^*} = (252.2 \pm 22.3_{(\text{OPE})} \pm 4_{(\text{syst})}) \text{ MeV}, \quad (4.1)$$

$$f_{D_s^*} = (305.5 \pm 26.8_{(\text{OPE})} \pm 5_{(\text{syst})}) \text{ MeV}, \quad (4.2)$$

and, for the various ratios of decay constants,

$$f_{D_s^*}/f_{D^*} = 1.211 \pm 0.061_{(\text{OPE})} \pm 0.007_{(\text{syst})}, \quad (4.3)$$

$$f_{D^*}/f_D = 1.221 \pm 0.080_{(\text{OPE})} \pm 0.008_{(\text{syst})}, \quad (4.4)$$

$$f_{D_s^*}/f_{D_s} = 1.241 \pm 0.057_{(\text{OPE})} \pm 0.007_{(\text{syst})}. \quad (4.5)$$

The OPE uncertainties in the decay constants of D^* and D_s^* and in the above ratios are, to large extent, due to the remaining dependence on the renormalization scale μ .

Our predictions agree well with those from lattice QCD, $f_{D^*} = (278 \pm 13 \pm 10) \text{ MeV}$ and $f_{D_s^*} = (311 \pm 9) \text{ MeV}$ [15].

Our results are in agreement with the recent estimates presented in Ref. [9], which also make use of our idea of a τ -dependent effective threshold. However, in our opinion, the estimates of [9] are not fully trustworthy: first, the OPE used in [9] contained errors which we correct (see (A.3) and (A.4)); second, the authors of [9] do not take properly into account the τ -dependence of the effective threshold when calculating the dual mass.

We stress that our algorithm for fixing τ -dependent effective thresholds allows us to provide, in addition to the OPE errors, also the systematic errors intrinsic to the QCD sum-rule technique. Although not entirely rigorous in the mathematical sense, our algorithm for obtaining the systematic errors has been verified in several examples within quantum mechanics, and proved to work well for decay constants of pseudoscalar mesons. The good news is that the systematic uncertainty turns out to be small and to be under control.

- (iv) The τ -dependent thresholds entail a visible shift in the sum-rule predictions for the decay constants of charmed vector mesons, increasing their numerical values by roughly 30 MeV compared to the outcomes when sticking to a constant threshold determined by the criterion of stability in the same Borel window.

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Appendix A: OPE for the vector correlator

The perturbative spectral densities have been calculated in three-loop order in [3] for one massless and one massive quark in terms of the pole mass M of the latter:

$$\rho_{\text{pert}}(s, M) = \rho^{(0)}(s, M) + a(\mu)\rho_1(s, M) + a^2(\mu)\rho_2(s, M), \quad a(\mu) \equiv \frac{\alpha_s(\mu)}{\pi}. \quad (A.1)$$

We reorganize this expansion in terms of the related running mass $m_Q \equiv \overline{m}_Q(\mu)$ (using the notations of [5]):

$$M = \frac{m_Q}{1 + a(\mu)r_m^{(1)} + a^2(\mu)r_m^{(2)}}. \quad (A.2)$$

The corresponding spectral densities and the expressions for the power corrections were taken from the Appendix to [9], except for Eqs. (A3) and (A4) therein, for which we obtain different results:

$$\Delta_1 \rho_T^{(\text{pert, NNLO})}(s) = -\frac{3}{8\pi^2} s z \left[(3 - 7z^2)r_m^{(1)2} - 2(1 - z^2)r_m^{(2)} \right], \quad (A.3)$$

$$\begin{aligned} \Delta_2 \rho_T^{(\text{pert, NNLO})}(s) = & -\frac{1}{16\pi^2} C_F r_m^{(1)} s \left[-12z(1 - z^2) (2\text{Li}_2(z) + \log(z) \log(1 - z)) \right. \\ & - 2z(9 + 6z - 17z^2) \log(z) \\ & \left. + 2(1 - z)(-4 + 5z + 17z^2) \log(1 - z) - z(1 - z)(17 + 15z) \right], \end{aligned} \quad (A.4)$$

$$z \equiv \frac{m_Q^2}{s}.$$

These equations replace the corresponding equations from [9].

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